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Influence of Refractive Index and Solar Concentration on Optical Power Absorption in Slabs

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Introduction

Since the late 1970's, NASA has been evaluating solar-pumped lasers for beamed power transmission in space. (See refs. 1 and 2.) Initially, efforts focused on gaseous lasing media because they facilitate cooling. More recently, solid and liquid media have been included in the research. Solid materials rely on heat conduction for cooling, and heat conduction is increased by geometries that feature large surface/volume ratios. Such a geometry is a slab. As presented here, a slab is a transparent material with two large, flat, parallel surfaces separated by a distance that is small relative to any other dimension. (The material may be solid, liquid, or gaseous—this report is quite general in its applicability—but the focus here (lasers) makes the results more important for solids.)

Practical solar-pumped lasers cannot operate on the normal power density (irradiance) provided by sunlight in near-Earth space (about 1365 W/m^2). The irradiance must be increased by concentration. That is, solar power must be collected over a large area and focused into a much smaller area that approximates the area of the laser. Typically, this is done with a reflective parabolic dish. The dish aperture is the large area. The small area is the image of the Sun at the focal distance. Average concentration is the ratio of the large area to the small area. Increasing aperture size by scaling all dimensions of the dish will not increase concentration because the focal area will increase proportionately. Concentration increases by extending the extremities of the dish while leaving the focal length unchanged. That is, concentration depends on the solid angle of incidence at the small focal area—not on the dish size.

The use of a parabolic dish concentrator to couple power to a slab laser introduces some interesting phenomena. The phenomena are caused by the refractive index of the slab and the reflector geometry required for solar concentration.

Any refractive index of slabs that mismatches that of free space causes reflections. Light is reflected from the surfaces, transmitted through the surfaces, and reflected and absorbed between the surfaces of the slab. Less light enters slab media (especially solids and liquids) because of reflections, but more of what enters is absorbed because of reflections.

Dish geometry necessarily introduces a solid angle of incidence on the slab and a variation of radiant power with angle of incidence. More power is incident at large angles of incidence, but more is reflected (compared with that at small angles of incidence).

Also, large angles of incidence create longer absorption paths in the solid.

A calculation that includes these phenomena is needed to determine power absorption. A separate calculation is made for light that is not incident through a solid angle (collimated light). A comparison of the two calculations reveals the influence of refractive index and solar concentration on absorbed power. The results of the two calculations can be applied to more than laser technology.

Analysis

In this analysis, a slab is located at the focal point of a parabolic dish concentrator. The plane of the slab is perpendicular to a line joining the focus and the vertex of the dish. The dish is uniformly irradiated over its aperture, except where the slab blocks 1 percent of the light (see the appendix) and all that radiation is incident on the slab. For the purposes of this calculation, the slab is large enough and thin enough that loss of radiation at its extremities is negligible.

At points on the slab, light is incident at angles from 0° to some large angle that depends on the focal length and diameter of the parabolic dish. In this geometry, radiation at small angles of incidence comes from more central regions of the dish aperture, whereas radiation at large angles of incidence comes from peripheral regions of the aperture. Since the peripheral regions of the aperture encompass more area than the central regions, more radiant power is incident at large angles of incidence than is incident at small angles. Also, at large angles of incidence, the absorption path length in the slab is increased. On the other hand, reflection at the slab surfaces tends to increase. From figure 1, the differential power incident on the slab is given by

$$\begin{aligned} dP &= I dA = 2\pi I r \sin \theta d(r \sin \theta) \\ &= 2\pi I r \sin \theta (r \cos \theta d\theta + \sin \theta dr) \end{aligned} \quad (1)$$

where

- I aperture irradiance
- A aperture area
- r distance of parabolic surface from focus
- θ angle of incidence

For a parabola,

$$r = \frac{R}{1 + \cos \theta}$$

$$dr = \frac{R \sin \theta d\theta}{(1 + \cos \theta)^2} \quad (2)$$

where R is the radius of the parabola for $\theta = 90^\circ$. Substituting equation (2) into equation (1) and integrating gives

$$P = 2\pi R^2 I \int_0^{\theta'} \left[\frac{\sin \theta \cos \theta}{(1 + \cos \theta)^2} + \left(\frac{\sin \theta}{1 + \cos \theta} \right)^3 \right] d\theta \quad (3)$$

where θ' is the maximum angle of incidence. Smaller diameter dishes with the same focal length subtend smaller angles. In that case the dish radius is expressed by

$$q = r \sin \theta' = \frac{R \sin \theta'}{1 + \cos \theta'}$$

$$R = \frac{q(1 + \cos \theta')}{\sin \theta'} \quad (4)$$

Hence, substituting equation (4) into equation (3) gives

$$P = 2\pi I A^2(\theta') q^2 \int_0^{\theta'} B(\theta) d\theta$$

$$A(\theta') = \frac{1 + \cos \theta'}{\sin \theta'}$$

$$B(\theta) = \frac{\sin \theta \cos \theta}{(1 + \cos \theta)^2} + \left(\frac{\sin \theta}{1 + \cos \theta} \right)^3$$

The portions of incident s and p polarized power absorbed by the slab are given by (ref. 3, pp. 36-41)

$$A_p = \frac{(1 - E_x) \left(1 - \frac{R_p}{2}\right)}{1 - E_x \frac{R_p}{2}} \quad (5)$$

and

$$A_s = \frac{(1 - E_x) \left(1 - \frac{R_s}{2}\right)}{1 - E_x \frac{R_s}{2}}$$

where

$$E_x = \exp \left(\frac{-n\sigma t}{\cos \phi} \right) = \exp \left(\frac{-\alpha}{\cos \phi} \right)$$

and

n	density of absorbing particles in slab
σ	absorption cross section of particle at a particular wavelength
t	thickness of slab
α	$= n\sigma t$
ϕ	$= \sin^{-1} \left(\frac{\sin \theta}{N} \right)$
N	index of refraction of slab at a particular wavelength
R_s, R_p	Fresnel power reflection coefficients at slab surfaces (parallel and perpendicular components of polarization)

Power absorbed in the slab then is given by

$$P_a = P(A_p + A_s)$$

Since A_p and A_s are functions of the angle of incidence, the portion of power incident on the dish aperture that is absorbed by the slab is given by

$$\frac{P_a}{P_{in}} = 2[A(\theta)]^2 \int_0^{\theta'} B(\theta) (A_p + A_s) d\theta \quad (6)$$

where

$$P_{in} = \pi q^2 I$$

is the power incident on the parabolic aperture. Solar concentration can be related to the solid angle subtended by the dish (if dish reflectivity is neglected) by the relation (ref. 4, p. 243)

$$C = \frac{\sin^2 \theta'}{a^2} \quad (7)$$

where a is the half-angle of solar subtense near Earth (approximately 0.25°). Equation (7) could be substituted into equation (6) to obtain the absorbed power as a function of solar concentration. The results are clearer however if P_a/P_{in} remains a function of the angle of incidence.

Results

Equation (6) was evaluated on a computer for various values of N , α , and θ' . The solid curves of figures 2 to 5 show the results for several real substances and are plotted to feature the effect of concentration (solid angle). Figure 2 depicts slabs that can be approximated by gases or with broadband antireflection coatings on solids. Results for a slab of water and some plastics are shown in figure 3. Figure 4 shows the results for most commonly encountered glasses

(index of refraction (N) of 1.5). Figure 5 shows the absorption at $N = 1.823$ of a Nd:YAG slab (i.e., a neodymium doped yttrium aluminum garnet slab). The data of figures 2 to 5 are plotted for half-degree increments of the abscissa. Some information on the variation of absorbed power with refractive index can be obtained by comparing figures 2 to 5. However, that information is plotted more explicitly in figure 6. The curves of figure 6 are the loci of the maxima of the solid curves in figures 2 to 5.

If the power incident on the slab were incident at only one angle instead of being distributed through a range of angles, the fraction of absorbed power is represented by the dotted curves on figures 2 to 5. The difference of the dotted- and solid-line curves represents the difference between absorption of concentrated light (incident at all angles up to θ') and absorption of collimated light (all of it incident at angle θ'). The comparison that best shows the effect of the solid angle of incidence, however, is the comparison between absorption of concentrated light (corresponding to a maximum angle θ') with that of collimated light at an angle of incidence of 0° . (In both of these cases, slab orientation is the same.)

Discussion

The influence of solar concentration is evident mainly at large values of concentration (large angles of incidence) in figures 2 to 5. At these large angles there is a slight increase in the fractional power absorptance over that at small angles of incidence. The solid angle of peak absorptance decreases approximately 20° in going from the less absorptive to the more absorptive slabs ($N > 1.0$). At very large solid angles absorbed power decreases somewhat, depending on the absorption coefficient. The decrease is caused primarily by increased reflection at grazing incidence, but it is counteracted to some degree by increased absorption path length in the slab and greater power collection at larger angles of incidence.

The most significant influence of refractive index is best shown by figure 6. There is a decrease in the fraction of absorbed power between indices of 1.0 and 1.1, and it is much more significant in the less absorptive slabs. In terms of materials, this means that solids and liquids absorb amounts of power (expressed as a fraction of incident power) that are comparable to each other and are significantly less than the fractional power absorption of gas and anti-reflection coated slabs (all having the same absorption coefficient). (Because of their density, however, thin gas slabs compare only with weakly absorbing, thin, solid or liquid slabs.)

Figure 2 represents the extreme case ($N = 1.0$) in which there is no reflection from the front surface of

the slab. Therefore, at grazing angles of incidence, absorption path lengths in the slab increase toward infinity and absorption approaches totality for collimated light. Concentrated light, conversely, never reaches total absorption because most of the rays are not at grazing incidence.

The absorption coefficient is usually dependent on the wavelength of light. Thus, these results apply strictly to light within a wavelength band where the absorption cross section of the absorbing species does not vary significantly. On the other hand, the results do not apply to coherent radiation (very narrow wavelength band) because the reflectance formulas apply only to incoherent light. To apply the results to wavelength bands that include a significant variation in absorption coefficient, summations of smaller bands must be made.

The index of refraction and the absorption coefficient are related through material permittivity such that at a wavelength where most absorption occurs, reflection maximizes also. In laser media the density of absorbing particles is usually small enough that the overall effect on the refractive index is small. In applications where the density of resonant absorbing particles is large, the absorption coefficient and the refractive index should both be measured at the same wavelength.

Concluding Remarks

For parabolic dish concentrators, the main effect of solar concentration is a slight increase in the fraction of absorbed power at large solid angles (maximum angles of incidence (θ') from 60° to 80°). At a given index of refraction, the solid angle of maximum absorptance decreases as absorption coefficient increases. The fraction of power absorbed from normally incident light ($\theta' = 0^\circ$) is a good approximation to the fraction absorbed through solid angles less than 2π steradians.

The influence of refractive index is primarily a reduction in the fraction of absorbed power with an increase in refractive index. The reduction is dramatic for weakly absorbing materials with a refractive index in the range from 1.0 to 1.1. For more absorptive materials, the variation is smaller and spreads over a larger range of refractive indices.

Calculations of absorbed power (expressed as a fraction of incident power) have been made for several representative materials and can be used to determine power absorption of slabs accurately if all the concentrated power is incident on the slab and spectral parameters are known at the wavelength of interest. If only a fraction of the concentrated power irradiates the slab, absorbed power must be reduced by that fraction.

Appendix

Solar Irradiance

The Sun provides a radiance of I watts per square meter of surface area per unit solid angle. Since each unit surface area radiates isotropically, the Sun may be treated as a radiating flat disk at distances far from it. In the vicinity of Earth, the power received from solar radiation is

$$P = I \times A_s \times \Omega$$

$$P = I \pi R_s^2 \frac{A}{R_\ell^2} = \left[\pi I \left(\frac{R_s}{R_\ell} \right)^2 \right] A$$

where

A_s area of solar disk, πR_s^2

R_s radius of solar disk

Ω solid angle subtended by area A near Earth, A/R_ℓ^2

R_ℓ distance from solar disk

Near Earth, the bracketed term is a fixed quantity called the solar constant. Thus, the power received from the Sun near Earth is proportional to the area that intercepts the sunlight.

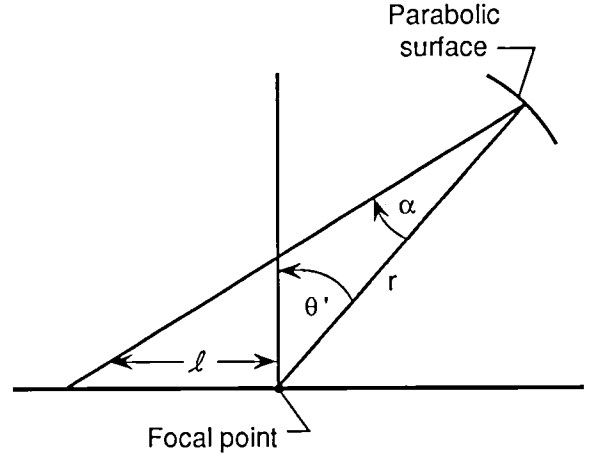
At any area element dA , solar power is received through the solid angle $\pi (R_s/R_\ell)^2$ and can be transmitted, reflected, and/or absorbed through the same solid angle. Any such area element on a parabolic dish reflects power through the same solid angle to the focal region.

At the focal region the radiation is received over an elliptical area. The farthest extent of the ellipse from the focal point of the parabolic dish is given by

$$\ell = \frac{r \sin \alpha}{\cos(\theta' + \alpha)}$$

and is illustrated in sketch A. Using equation (2) of the text gives

$$\ell = \frac{q \sin \alpha}{\cos(\theta' + \alpha) \sin \theta'}$$



Sketch A

The distance ℓ determines the radius of a slab that can intercept all radiation reflected from the dish. If the area of the slab is arbitrarily set to 1 percent of the aperture area of the dish, then

$$\frac{q}{\ell} = \frac{\cos(\theta' + \alpha) \sin \theta'}{\sin \alpha} = 10$$

$$\alpha = 0.25^\circ \text{ and } \theta' = 87.24^\circ$$

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3. Bohren, Craig F.; and Huffman, Donald R.: *Absorption and Scattering of Light by Small Particles*. John Wiley & Sons, Inc., c.1983.
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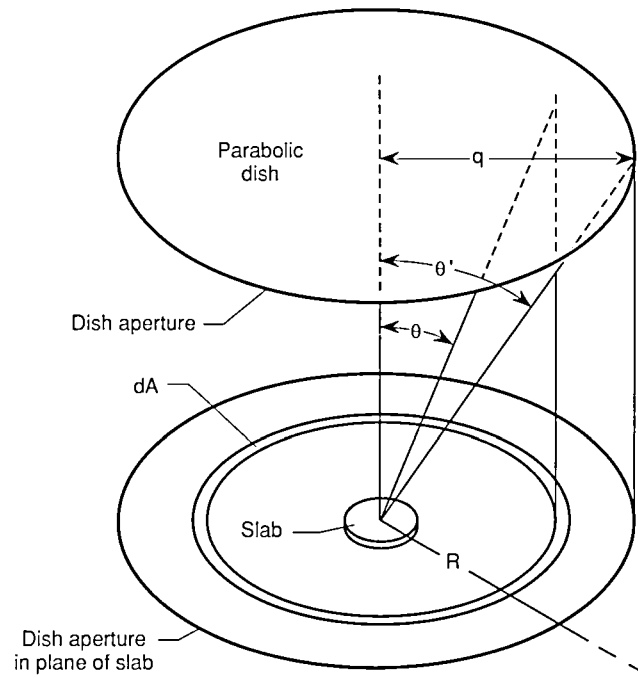


Figure 1. Geometric variables.

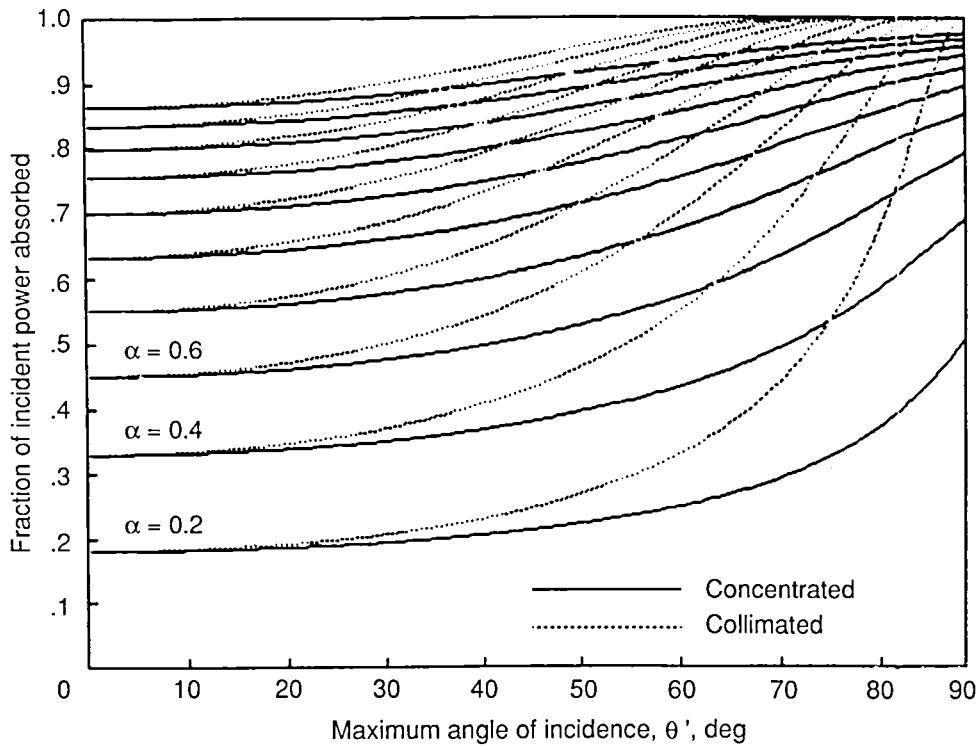


Figure 2. Optical power absorbed in coated slab. $N = 1.0$.

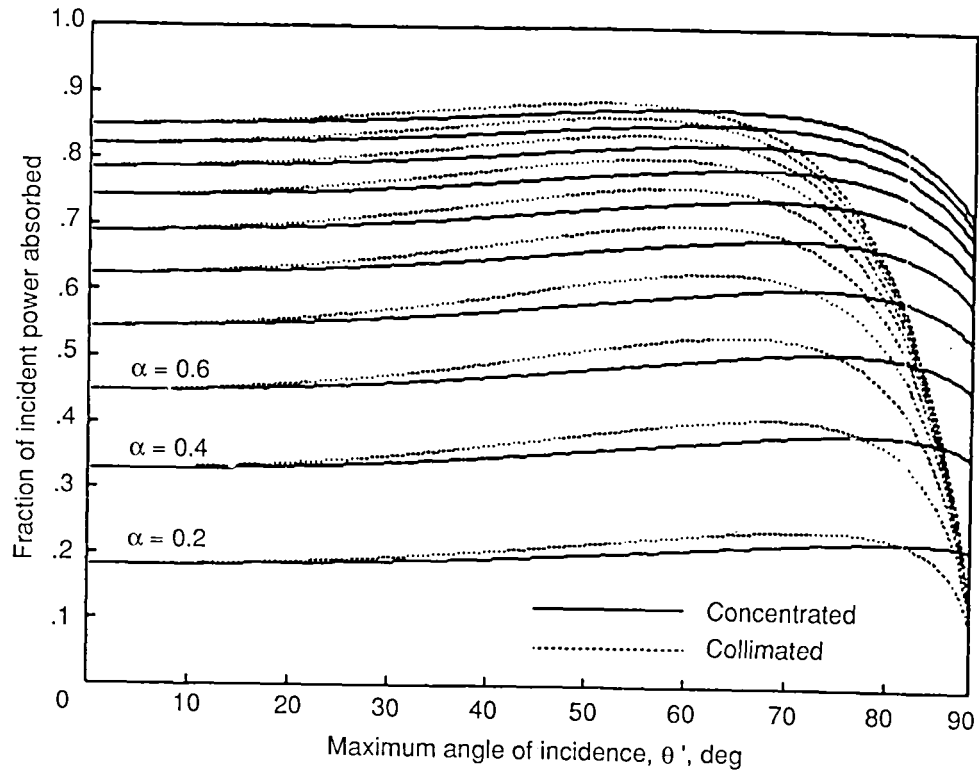


Figure 3. Optical power absorbed in liquid slab. $N = 1.32$.

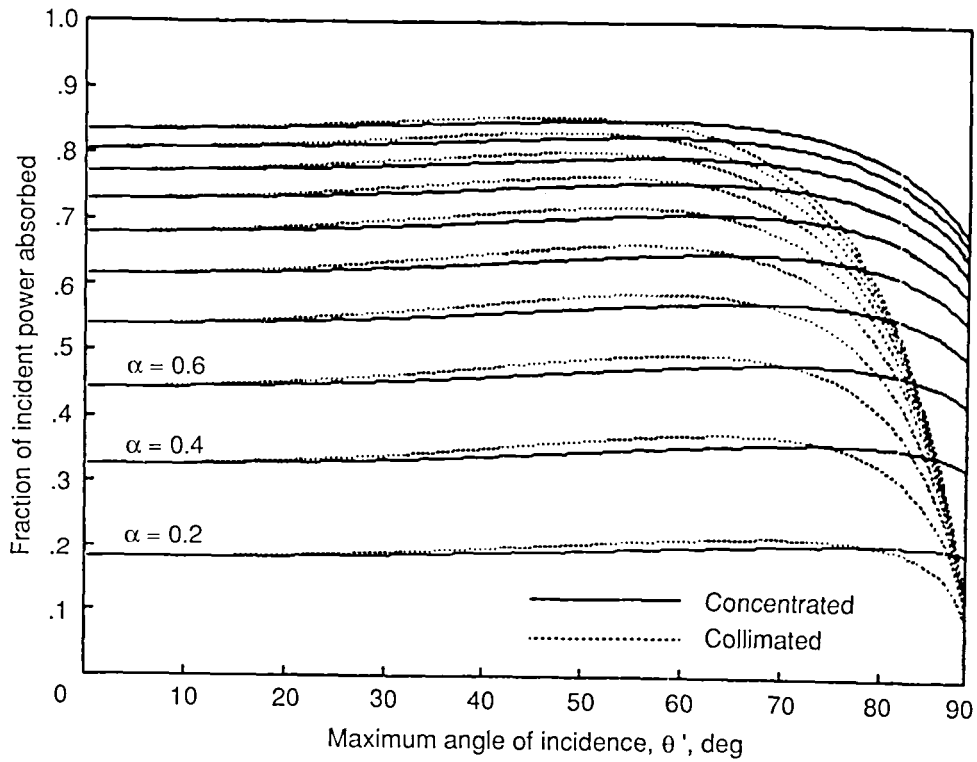


Figure 4. Optical power absorbed in glass slab. $N = 1.5$.

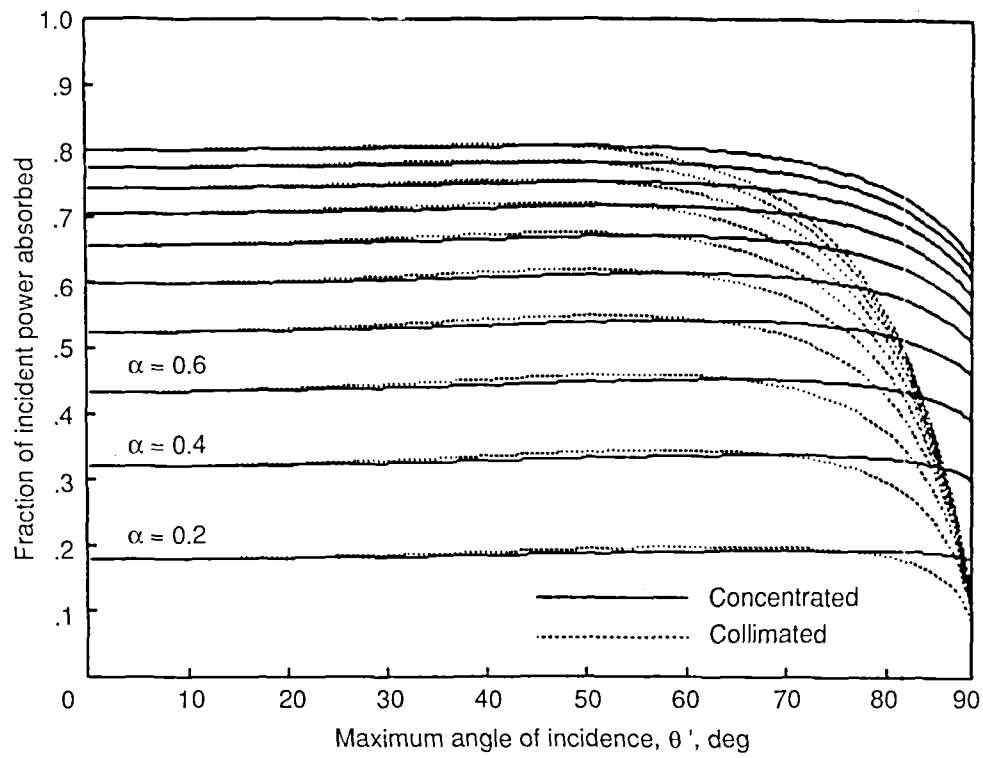


Figure 5. Optical power absorbed in Nd:YAG slab. $N = 1.823$.

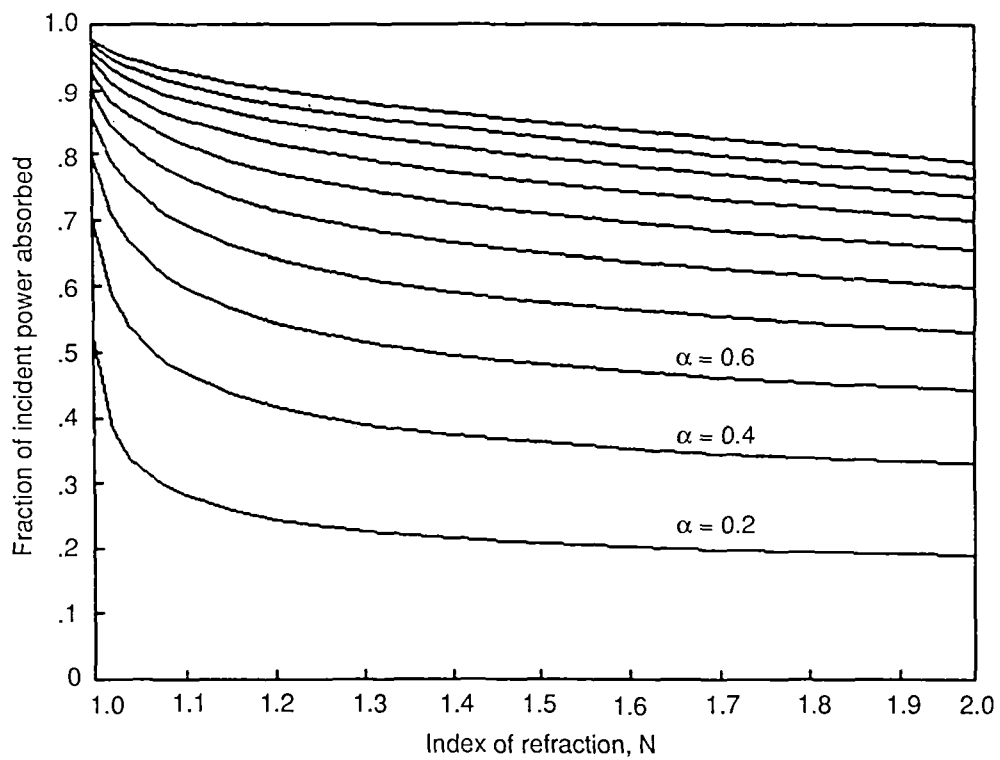


Figure 6. Optical power absorbed from concentrated light.



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16. Abstract The optical power absorbed by a slab at the focus of a parabolic dish concentrator is calculated as a fraction of the power incident on the dish. The calculations are plotted versus maximum angle of incidence of irradiation (which corresponds to different magnitudes of solar concentration) with absorption coefficient as a parameter for several different indices of refraction that represent real materials. Results show that the effect of the solid angle of incidence is a small increase in the fraction of absorbed power over that of a similarly oriented slab irradiated by collimated light. Large indices of refraction allow less fractional absorptance and affect the solid angle at which maximum fractional absorptance occurs.					
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